Published by Institute of Physics Publishing for SISSA
Received: *October 12, 2005*
Accepted: *November 8, 2005*
Published: *November 28, 2005*

Supersymmetric D-branes and calibrations on general $\mathcal{N} = 1$ backgrounds

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Abstract: We study the conditions to have supersymmetric D-branes on general $\mathcal{N} = 1$ backgrounds with Ramond-Ramond fluxes. These conditions can be written in terms of the two pure spinors associated to the $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure on $T_M \oplus T^{\star}_M$, and can be split into two parts each involving a different pure spinor. The first involves the integrable pure spinor and requires the D-brane to wrap a generalised complex submanifold with respect to the generalised complex structure associated to it. The second contains the non-integrable pure spinor and is related to the stability of the brane. The two conditions can be rephrased as a generalised calibration condition for the brane. The results preserve the generalised mirror symmetry relating the type IIA and IIB backgrounds considered, giving further evidence for this duality.

Keywords: D-branes, Flux compactifications.

JHEP11(2005)048

Contents

1. Introduction

The study of string and supergravity backgrounds with fluxes has received much attention in the recent years due to the key role that they play in many interesting situations. For example, they appear to be fundamental in the search for more realistic and phenomenologically interesting stringy scenarios, and also in the construction of string models holographically dual to relevant gauge theories. As fundamental objects of the theory, D-branes occupy a preeminent position in all these models and several aspects of their physics in such nontrivial situations deserve a better understanding.

In this paper we aim to study the geometry of supersymmetric D-brane configurations in a very general class of supergravity backgrounds preserving four-dimensional Poincaré invariance and $\mathcal{N}=1$ supersymmetry. Such backgrounds correspond to warped products of the four-dimensional Minkowski space-time and an internal six-dimensional manifold M with general fluxes turned on. $\mathcal{N}=1$ supersymmetry requires the existence of four independent 10d killing spinors, whose most general form can be written in terms of two internal six-dimensional Weyl spinors $\eta_+^{(1)}$ and $\eta_+^{(2)}$. This implies that M has a reduced $\mathrm{SU}(3)\times\mathrm{SU}(3)$ structure on $T_M\oplus T^\star_M$, which may be further restricted to a SU(3) or SU(2) structure on T_M. As discussed in [1, 2], these vacua can be nicely characterised in terms of two $O(6,6)$ *pure spinors* $\Psi^\pm=\eta_+^{(1)}\otimes\eta_\pm^{(2)}$. Using the Clifford map, the pure spinors can be equivalently seen as formal sums of forms $\Psi^\pm=\sum_k\Psi^\pm_{(k)}$, where k is even for Ψ^+ and odd for Ψ^-.

This formalism introduces a natural relation to generalised complex geometry [3, 4].[1] The two pure spinors are associated to *generalised almost complex structures* whose (generalised) integrability corresponds in turn to 'closureness' of the pure spinors under the twisted derivative operator $d_H = d + H\wedge$. In [2] it has been shown that the supersymmetry conditions provide the integrability of the almost complex structure associated to one pure spinor and that it defines a twisted generalised Calabi-Yau (CY) structure à la Hitchin [3] on the internal manifold. On the other hand, the second pure spinor is not integrable due to the presence of Ramond-Ramond (RR) field-strengths which act as an obstruction to integrability. As a consequence, if for example we restrict ourselves to the SU(3) case, the internal manifold will be either symplectic (IIA) or complex (IIB). In the more general SU(3) $\times$ SU(3) case, the manifold is a complex-symplectic hybrid, even if IIA and IIB continue to "prefer" symplectic and complex manifolds respectively [2].

In the following sections we will see how it is possible to characterise the supersymmetric D-brane configurations completely in terms of the two pure spinors for this general class of $\mathcal{N} = 1$ backgrounds. We will mainly focus on the case of branes filling the flat 4d space-time and the resulting equations (see equations (3.14) and (3.15) or equivalently (3.16) and (3.17)) represent the generalisation to $\mathcal{N} = 1$ flux backgrounds of the conditions obtained in [6, 7] for branes wrapped on cycles of CY_3. This can be seen from the form these conditions take once we restrict to the SU(3) case (see equations (5.4) and (5.5) or equivalently (5.6) and (5.7)), which can be considered as formally the closest to the CY case.[2] One preliminary necessary requirement in order to get supersymmetric branes is that the two internal spinors $\eta^{(1)}$ and $\eta^{(2)}$ must have the same norm. Then, the supersymmetry conditions split into two parts involving the two pure spinors $\Psi^{\pm}$ and are completely symmetric under the exchange $\Psi^+ \leftrightarrow \Psi^-$ as one goes from type IIA backgrounds to type IIB and vice-versa. This symmetry can be seen as a generalisation of the usual mirror symmetry between supersymmetric cycles on standard CY's.

The first supersymmetry condition for a space-time filling D-brane wrapping an internal n-cycle can be written in the form

$$\Big\{P[(g^{mk}\imath_k + dx^m\wedge)\Psi]\wedge e^{\mathcal{F}}\Big\}_{(n)} = 0\,, \tag{1.1}$$

where $\mathcal{F} = f + P[B]$ (f is the world-volume field-strength), Ψ is equal to Ψ^- in IIB and Ψ^+ in IIA, $P[.]$ indicates the pullback on the worldvolume of the brane, and in the left hand side we consider only forms of rank equal to the dimension n of the wrapped cycle. These pure spinors are exactly the integrable ones for each case and we will discuss how this condition means that supersymmetric cycles are generalised complex submanifolds with respect to the appropriate integrable generalised complex structure $\mathcal{J}$, as defined in [4]. Then, supersymmetric branes wrap an appropriate generalisation of a complex submanifold in type IIB and of coisotropic submanifolds in type IIA, and this identification becomes

[1] See [5] for previous discussions on the use of SU(3) $\times$ SU(3) and other "generalised" structures to describe supersymmetric type II compactifications in the context of generalised geometries.

[2] Equivalent conditions have recently been presented for D-branes on IIB SU(3)-structure backgrounds in [8], where several interesting applications to the warped Calabi-Yau subcase [9] are also discussed.

precise in the SU(3)-structure case. This result is completely analogous to the one recently discussed in [10] where D-branes on supersymmetric backgrounds with only nontrivial Neveu-Schwarz (NS) fields are considered (for previous work on branes in the context of generalised complex geometry see [11 − 15]).

The second supersymmetry condition is related to the stability of the D-brane and can be written as

$$\big\{\mathrm{Im}\,(iP[\Psi])\wedge e^{\mathcal{F}}\big\}_{(n)}=0\,, \tag{1.2}$$

where now Ψ is equal to Ψ^+ in IIB and Ψ^- in IIA (i.e. is the non-integrable pure spinor). The two conditions (1.1) and (1.2) imply that for a suitable choice of orientation on the wrapped cycle, the D-brane configuration is supersymmetric. Since we are considering backgrounds with nontrivial RR fluxes turned on, reversing the orientation on the brane does not generally preserve supersymmetry.

The above two conditions can be rephrased in terms of a single condition which also encodes the necessary orientation requirement. For a D-brane wrapping an internal n-cycle, this is given by

$$\big\{\mathrm{Re}\,(-iP[\Psi])\wedge e^{\mathcal{F}}\big\}_{(n)}=\frac{||\Psi||}{8}\sqrt{-\det(g+\mathcal{F})}d\sigma^1\wedge\cdots d\sigma^n\,, \tag{1.3}$$

where again Ψ is equal to Ψ^+ in IIB and Ψ^- in IIA and $||\Psi||^2=\mathrm{Tr}(\Psi\Psi^\dagger)$. This condition will be identified as a calibration condition with respect to an appropriate *generalised calibration* $\omega=\sum_k\omega_{(k)}$, with $\omega_{(k)}$ being a k form, which by definition is twisted closed, i.e. $d_H\omega=0$, and must fulfil a condition of minimisation of the D-brane energy density. More specifically, for any space-time filling D-brane wrapping any internal cycle Σ and with any worldvolume field strength $\mathcal{F}$ (such that $d\mathcal{F}=P_\Sigma[H]$), we must have

$$P_\Sigma[\omega]\wedge e^{\mathcal{F}}\leq\mathcal{E}(\Sigma,\mathcal{F})\,, \tag{1.4}$$

where $\mathcal{E}$ represents the energy density (see equation (6.9)) and in the left hand side we mean that only forms of rank equal to the dimension of the wrapped cycle are considered. An analogous definition of generalised calibration has recently been used in [10] for the case with only nontrivial NS fields, and our result represents an extension of that proposal in presence of non-zero RR fluxes.

The paper is organised as follows. In section 2 we review the basic conditions defining the general class of $\mathcal{N}=1$ backgrounds we are considering. In section 3 we derive the supersymmetry conditions for supersymmetric D-branes using κ-symmetry and express them in terms of the pure spinors $\Psi^\pm$ characterising our backgrounds. In section 4 and 5 we clarify the meaning of the conditions for the internal supersymmetric cycles, identifying them as generalised complex submanifold calibrated with respect to the appropriate definition of generalised calibration. Finally we present our conclusions. Appendix A contains some basic properties of the almost complex structure and (3,0)-form that can be constructed from an internal spinor. Appendix B presents some details on the calculation of the background supersymmetry conditions needed in our analysis.

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2. Basic results on $\mathcal{N}=1$ vacua

We are interested in type II warped backgrounds preserving four-dimensional Poincaré invariance and $\mathcal{N}=1$ supersymmetry, with the most general fluxes and fields turned on. The ansatz for the ten dimensional metric G_{MN} is

$$ds^2 = e^{2A(y)} dx^\mu dx_\mu + g_{mn}(y) dy^m dy^n \,, \tag{2.1}$$

where x^μ, $\mu = 0, \ldots, 3$ label the four-dimensional flat space, and y^m, $m = 1, \ldots, 6$, the internal space. Let us introduce the modified RR field strengths

$$F_{(n+1)} = dC_{(n)} + H \wedge C_{(n-2)} \,, \tag{2.2}$$

where $dC_{(n)}$ are the standard RR field strengths.[3] In order to preserve four dimensional Poincaré invariance we can write

$$F_{(n)} = \hat{F}_{(n)} + \mathrm{Vol}_{(4)} \wedge \tilde{F}_{(n-4)} \,. \tag{2.3}$$

The relation $F_{(n)} = (-)^{\frac{(n-1)(n-2)}{2}} \star_{10} F_{(10-n)}$ between the lower and higher rank field strengths translates into a relation of the form $\tilde{F}_{(n)} = (-)^{\frac{(n-1)(n-2)}{2}} \star_6 \hat{F}_{(6-n)}$ between their internal components. The ten dimensional gamma matrices $\Gamma_{\underline{M}}$ (underlined indices correspond to flat indices) can be chosen in a real representation and decomposed in the following way

$$\Gamma_{\underline{\mu}} = \gamma_{\underline{\mu}} \otimes 1 \,, \qquad \Gamma_{\underline{m}} = \gamma_{(4)} \otimes \hat{\gamma}_{\underline{m}} \,, \tag{2.4}$$

where the four-dimensional gammas $\gamma_{\underline{\mu}}$ are real and the six-dimensional ones $\hat{\gamma}_{\underline{m}}$ are anti-symmetric and purely imaginary. The four- and six-dimensional chirality operators are given respectively by

$$\gamma_{(4)} = i\gamma^{\underline{0123}} \,, \qquad \hat{\gamma}_{(6)} = -i\hat{\gamma}^{\underline{123456}} \,, \tag{2.5}$$

so that the 10d chirality operator can be written as $\Gamma_{(10)} = \Gamma^{\underline{0\cdots 9}} = \gamma_{(4)} \otimes \hat{\gamma}_{(6)}$.

For type IIA backgrounds the supersymmetry parameter is a 10d Majorana spinor ε that can be split in two Majorana-Weyl (MW) spinors of opposite chirality:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \,, \qquad \Gamma_{(10)} \varepsilon_1 = \varepsilon_1 \,, \qquad \Gamma_{(10)} \varepsilon_2 = -\varepsilon_2 \,. \tag{2.6}$$

Since we are interested only in four-dimensional $\mathcal{N}=1$ backgrounds, they must have 4 independent Killing spinors that can be decomposed as

$$\begin{aligned} \varepsilon_1(y) &= \zeta_+ \otimes \eta_+^{(1)}(y) + \zeta_- \otimes \eta_-^{(1)}(y) \,, \\ \varepsilon_2(y) &= \zeta_+ \otimes \eta_-^{(2)}(y) + \zeta_- \otimes \eta_+^{(2)}(y) \,, \end{aligned} \tag{2.7}$$

where ζ_+ is a generic constant four-dimensional spinor of positive chirality, while the $\eta_+^{(a)}$ are two particular six-dimensional spinor fields of positive chirality that characterise the solution and

$$\zeta_- = (\zeta_+)^* \,, \qquad \eta_-^{(a)} = \left(\eta_+^{(a)}\right)^* \,. \tag{2.8}$$

[3] We will essentially follow the conventions of [1, 2], up to some differences consisting in a sign for H in type IIB and the sign change $C_{(2n+1)} \to (-)^n C_{(2n+1)}$ in type IIA.

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In type IIB the two supersymmetry parameters $\varepsilon_{1,2}$ are MW real spinors of positive 10d chirality ($\Gamma_{(10)}\varepsilon_{1,2} = \varepsilon_{1,2}$). In this case

$$\varepsilon_a(y) = \zeta_+ \otimes \eta_+^{(a)}(y) + \zeta_- \otimes \eta_-^{(a)}(y)\,, \tag{2.9}$$

where again $\zeta_- = (\zeta_+)^*$ and $\eta_-^{(a)} = (\eta_+^{(a)})^*$. The existence of the internal spinors $\eta_+^{(1)}$ and $\eta_+^{(2)}$ associated to these $\mathcal{N}=1$ backgrounds generally specifies an SU(3) $\times$ SU(3)-structure on $T_M \oplus T_M^\star$.

As discussed in [2], in order to analyse the supersymmetric conditions for the background, it is convenient to use the bispinor formalism. Any $O(6,6)$ bispinor $\not\chi$ can be written as a sum of antisymmetric products of gamma matrices

$$\not\chi = \sum_k \frac{1}{k!}\chi^{(k)}_{m_1\ldots m_k}\hat{\gamma}^{m_1\ldots m_k}\,, \tag{2.10}$$

which, via the Clifford map, is in one-to-one correspondence with the formal sum of forms of different degree

$$\chi = \sum_k \frac{1}{k!}\chi^{(k)}_{m_1\ldots m_k} dx^{m_1}\wedge\cdots\wedge dx^{m_k}\,. \tag{2.11}$$

We can then associate two pure spinors to our internal spinors $\eta_+^{(1)}$ and $\eta_+^{(2)}$

$$\not\Psi^+ = \eta_+^{(1)} \otimes \eta_+^{(2)\dagger}\,, \qquad \not\Psi^- = \eta_+^{(1)} \otimes \eta_-^{(2)\dagger} \tag{2.12}$$

corresponding to sums of forms of definite parity

$$\Psi^+ = \sum_{k\geq 0}\Psi^+_{(2k)}\,, \qquad \Psi^- = \sum_{k\geq 0}\Psi^-_{(2k+1)}\,. \tag{2.13}$$

Following [2], we also define

$$||\eta^{(1)}||^2 = |a|^2\,, \qquad ||\eta^{(2)}||^2 = |b|^2\,. \tag{2.14}$$

Using the Clifford map, it is possible to use the gravitino and dilatino Killing conditions to compute $d\Psi^\pm$ [2]. The resulting equations are

$$\begin{aligned} e^{-2A+\Phi}(d+H\wedge)[e^{2A-\Phi}\Psi_1] &= dA\wedge\bar{\Psi}_1 + \frac{e^\Phi}{16}[(|a|^2-|b|^2)\hat{F} + i(|a|^2+|b|^2)\tilde{F}]\,, \\ (d+H\wedge)[e^{2A-\Phi}\Psi_2] &= 0\,, \end{aligned} \tag{2.15}$$

where for type IIA we have

$$\Psi_1 = \Psi^-\,, \qquad \Psi_2 = \Psi^+ \qquad \text{and} \qquad F = F_A = F_{(0)} + F_{(2)} + F_{(4)} + F_{(6)}\,, \tag{2.16}$$

while for type IIB

$$\Psi_1 = \Psi^+\,, \qquad \Psi_2 = \Psi^- \qquad \text{and} \qquad F = F_B = F_{(1)} + F_{(3)} + F_{(5)}\,. \tag{2.17}$$

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Note that, taking into account the different conventions, the first of (2.15) has some sign differences with equations (3.2) and (3.3) of [2].[4] For this reason we give some details of the computations leading to (2.15) in appendix B. The second condition means that the generalised almost complex structure associated to Ψ_2 is integrable while in the first condition the RR fields represent an obstruction to the integrability of the generalised almost complex structure associated to Ψ_1. Using the gravitino Killing equations one can furthermore show that

$$d|a|^2 = |b|^2 dA\,, \qquad d|b|^2 = |a|^2 dA\,. \tag{2.18}$$

As discussed in [2], it can be proven that equations (2.15) and (2.18) are completely equivalent to the full set of supersymmetric Killing conditions and then can be considered as necessary and sufficient conditions to have a supersymmetric background. Furthermore, one has to bear in mind that these equations only make sense if not all of the RR field strengths are vanishing and that in order to have a complete supergravity solution one has to supplement these conditions with the Bianchi identities and the equations of motion for the fluxes [16].

The supersymmetry conditions (2.15) and (2.18) are identical in form for type IIA and IIB and the two cases are exactly related by the exchange

$$\Psi^+ \leftrightarrow \Psi^- \qquad \text{and} \qquad F_A \leftrightarrow F_B\,. \tag{2.19}$$

This relation can be seen as a generalised mirror symmetry for type II backgrounds with SU(3) × SU(3) structure and, as we will see, the conditions for having supersymmetric branes respect this symmetry, giving further evidence for it. For further discussions on generalised mirror symmetry, see e.g. [17–22].

Let us finally remember that these backgrounds contain as subcases the SU(3) and SU(2) structure backgrounds. In the SU(3) case we have to require the two $\eta_+^{(a)}$ to be linearly dependent, i.e. $\eta_+^{(1)} = a\eta_+$ and $\eta_+^{(2)} = b\eta_+$ for a given six-dimensional spinor field η_+, with $\eta_+^\dagger \eta_+ = 1$. On the other hand we have SU(2)-structure when $\eta_+^{(1)}$ and $\eta_+^{(2)}$ are never parallel. We refer the reader to the detailed discussion of these cases given in [2].

3. Supersymmetric D-branes on $\mathcal{N} = 1$ vacua

In general a Dp-brane configuration is defined by the embedding $\sigma^\alpha \mapsto (x^\mu(\sigma), y^m(\sigma))$, $\alpha = 0, \ldots, p$ and preserves a given supersymmetry ε of the background if it satisfies the condition

$$\bar{\varepsilon}\Gamma_{Dp} = \bar{\varepsilon}\,, \tag{3.1}$$

where Γ_{Dp} is the worldvolume chiral operator entering the κ-symmetry transformations [23, 24]. It is convenient to use a double spinor convention for both type IIB and type IIA where in this last case the two spinors of opposite chirality are organised in a two component vector. Using the explicit form of the κ-operators in this notation,[5] the supersymmetry

[4] *Note added*: we thank the authors of [2] for private communications confirming the sign mistakes appearing in equations (3.2) and (3.3) in the original version of their paper.

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condition reduces to

$$\hat{\Gamma}_{Dp}\varepsilon_2 = \varepsilon_1 \,, \tag{3.2}$$

where

$$\hat{\Gamma}_{Dp} = \frac{1}{\sqrt{-\det(P[G]+\mathcal{F})}} \sum_{2l+s=p+1} \frac{\epsilon^{\alpha_1\ldots\alpha_{2l}\beta_1\ldots\beta_s}}{l!s!2^l} \mathcal{F}_{\alpha_1\alpha_2}\cdots\mathcal{F}_{\alpha_{2l-1}\alpha_{2l}}\Gamma_{\beta_1\ldots\beta_s} \tag{3.3}$$

and $\hat{\Gamma}^{-1}_{Dp}(\mathcal{F}) = (-)^{\mathrm{Int}[\frac{p+3}{2}]}\hat{\Gamma}_{Dp}(-\mathcal{F})$. Let us start by restricting our attention to Dp-branes filling the time plus q flat directions (with no worldvolume flux in these directions), and wrapping an internal $(p-q)$-cycle. We can then decompose the above operators into four- and six-dimensional components as follows

$$\hat{\Gamma}_{Dp} = \gamma_{\underline{0\ldots q}}\gamma_{(4)}^{p-q} \otimes \hat{\gamma}'_{(p-q)} \,, \tag{3.4}$$

where

$$\hat{\gamma}'_{(r)} = \frac{1}{\sqrt{\det(P[g]+\mathcal{F})}} \sum_{2l+s=r} \frac{\epsilon^{\alpha_1\ldots\alpha_{2l}\beta_1\ldots\beta_s}}{l!s!2^l} \mathcal{F}_{\alpha_1\alpha_2}\cdots\mathcal{F}_{\alpha_{2l-1}\alpha_{2l}}\hat{\gamma}_{\beta_1\ldots\beta_s} \,, \tag{3.5}$$

is a unitary operator acting on the internal spinors.

By considering general Dp-branes in both type IIA/IIB backgrounds and using (2.7), (3.3) and (3.4), it is possible to see that the supersymmetric condition (3.1) can be split into the four-dimensional condition

$$\gamma_{\underline{0\ldots q}}\zeta_+ = \alpha^{-1}\zeta_{(-)^{q+1}} \,, \tag{3.6}$$

and the internal six-dimensional one

$$\hat{\gamma}'_{(p-q)}\eta^{(2)}_{(-)^{p+1}} = \alpha\eta^{(1)}_{(-)^{q+1}} \,. \tag{3.7}$$

By consistency with the complex conjugate of these expressions and the fact that $\gamma^2_{\underline{0\ldots q}} = -(-)^{\frac{q(q+1)}{2}}$, it can be seen that the case $q=0$, i.e. the case where we have an effective four-dimensional particle, can never be supersymmetric, while for $q=1,2,3$ one has the condition that $\alpha = e^{i\theta}$, i.e. α is a pure phase. More explicitly $\theta = 0$ or π for $q=1$ (effective string), θ is arbitrary for $q=2$ (domain-wall) and $\theta = -\pi/2$ for $q=3$ (space-time filling branes). From the unitarity of the operator $\hat{\gamma}'_{(r)}$, it also follows that we must have the following constraints on the internal spinors

$$||\eta^{(1)}||^2 = ||\eta^{(2)}||^2 \,, \tag{3.8}$$

and from (2.18) we then see that once the condition (3.8) is fulfilled at one point for our backgrounds , it is automatically valid at all points.

For the purposes of this work we are interested in spacetime filling branes and hence from this point on we shall consider only these cases. Supersymmetry conditions for the other cases listed above are easily found by reinstating θ-dependence in the appropriate way.

[5] Here we use the κ-symmetry operators constructed from T-duality in [25], which are identical to those given in [24] up to some different overall signs. Their explicit form in double spinor notation in both IIA and IIB can be found in [26].

In the case of four-dimensional space-time filling branes, the four-dimensional condition is automatically satisfied once we set $\theta = -\pi/2$ and one is left with the following internal conditions

$$\begin{cases} i\hat{\gamma}'_{(2k)}\eta_+^{(2)} = \eta_+^{(1)}\,, & \text{in IIB}\,, \\ i\hat{\gamma}'_{(2k+1)}\eta_+^{(2)} = \eta_-^{(1)}\,, & \text{in IIA}\,. \end{cases} \tag{3.9}$$

We would like now to write the supersymmetry conditions (3.9) in terms of the geometrical objects Ψ^+ and Ψ^- introduced in section 2. In order to do this, it is useful to decompose the spinorial quantities entering (3.9) in the basis defined by

$$\eta_+^{(1)} \quad , \quad \eta_-^{(1)}\,, \qquad \hat{\gamma}_m\eta_+^{(1)} \qquad \text{and} \qquad \hat{\gamma}_m\eta_-^{(1)}\,. \tag{3.10}$$

By decomposing the supersymmetry conditions (3.9) in this basis, one obtains a set of equations written in a more geometric fashion in terms of the pull-back to the worldvolume of Ψ^+ and Ψ^-. Explicitly, for even $2k$-cycles we have the conditions

$$\begin{aligned} &\Big\{P[\Psi^+]\wedge e^{\mathcal{F}}\Big\}_{(2k)} = \frac{i|a|^2}{8}\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1\wedge\cdots\wedge d\sigma^{2k}\,, \\ &\Big\{P[dx^m\wedge\Psi^- + g^{mn}\imath_n\Psi^-]\wedge e^{\mathcal{F}}\Big\}_{(2k)} = 0\,, \end{aligned} \tag{3.11}$$

while for odd $(2k+1)$-cycles we have

$$\begin{aligned} &\Big\{P[\Psi^-]\wedge e^{\mathcal{F}}\Big\}_{(2k+1)} = \frac{i|a|^2}{8}\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1\wedge\cdots\wedge d\sigma^{2k+1}\,, \\ &\Big\{P[dx^m\wedge\Psi^+ + g^{mn}\imath_n\Psi^+]\wedge e^{\mathcal{F}}\Big\}_{(2k+1)} = 0\,. \end{aligned} \tag{3.12}$$

Note that these equations are identical if we interchange

$$\Psi^+ \leftrightarrow \Psi^-\,. \tag{3.13}$$

They then respect the generalised mirror symmetry (2.19) that relates the type IIA and IIB $\mathcal{N}=1$ supersymmetric backgrounds we are considering.

In the following section we will discuss the geometrical interpretation of the supersymmetric conditions (3.11) and (3.12). As a preliminary step, it is useful to observe that they are not independent. Indeed, we obtained these conditions by expanding (3.9) in the basis (3.10) and then, using the unitarity of $\hat{\gamma}'(\mathcal{F})$, it is easy to see that the first equations of (3.11) and (3.12) imply the seconds. Viceversa, the second conditions determine the first up to an overall arbitrary (in general, point dependent) phase. Moreover, once again using the unitarity of $\hat{\gamma}'(\mathcal{F})$, the first conditions can be furthermore restricted in such a way that we can characterise the supersymmetry cycles in the following way:

$$\begin{aligned} \Big\{\mathrm{Im}\,(iP[\Psi^+])\wedge e^{\mathcal{F}}\Big\}_{(2k)} &= 0\,, \\ \Big\{P[dx^m\wedge\Psi^- + g^{mn}\imath_n\Psi^-]\wedge e^{\mathcal{F}}\Big\}_{(2k)} &= 0\,, \end{aligned} \tag{3.14}$$

for even $2k$-cycles, while

$$\begin{aligned}
\Big\{ \mathrm{Im}\,(iP[\Psi^-]) \wedge e^{\mathcal{F}} \Big\}_{(2k+1)} &= 0\,, \\
\Big\{ P[dx^m \wedge \Psi^+ + g^{mn}\imath_n \Psi^+] \wedge e^{\mathcal{F}} \Big\}_{(2k+1)} &= 0\,,
\end{aligned} \tag{3.15}$$

for odd $(2k+1)$-cycles. Note that these conditions do not strictly speaking imply that the wrapping brane is supersymmetric but in general it is supersymmetric only for one orientation. If the RR fields were turned off, the orientation would be arbitrary because a change of orientation would amount in considering an anti D-brane instead of a D-brane or viceversa, and these feel the background fields in the same way. However, we are considering the case with nontrivial RR fields. D-branes and anti D-branes then react to the background in a different way and the orientation cannot be ignored, meaning that the conditions given in (3.14) and (3.15) are in fact necessary and sufficient only for the brane to admit at least an orientation making it supersymmetric.

The above conditions can be substituted by the following single condition that encodes also the necessary orientation requirement:

$$\Big\{ \mathrm{Re}\,(-iP[\Psi^+]) \wedge e^{\mathcal{F}} \Big\}_{(2k)} = \frac{|a|^2}{8}\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1 \wedge \cdots \wedge d\sigma^{2k}\,, \tag{3.16}$$

for even $2k$-cycles, while for odd $(2k+1)$-cycles

$$\Big\{ \mathrm{Re}\,(-iP[\Psi^-]) \wedge e^{\mathcal{F}} \Big\}_{(2k+1)} = \frac{|a|^2}{8}\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1 \wedge \cdots \wedge d\sigma^{2k+1}\,. \tag{3.17}$$

Note that since we are assuming that the internal spinors have the same norm, in the above expressions we can write $|a|^2$ in terms of any of the two pure spinors as follows

$$|a|^4 = ||\Psi||^2 = \mathrm{Tr}(\not{\Psi}\not{\Psi}^\dagger) = 8\sum_k |\Psi_{(k)}|^2\,. \tag{3.18}$$

We will see in section 6 that we can interpret the equations (3.16) and (3.17), and then also (3.14) and (3.15) plus an appropriate choice of the orientation, as generalised calibration conditions.

4. The geometry of the supersymmetric D-branes

We shall now discuss the geometrical meaning of the second conditions of (3.14) and (3.15). As we will see, supersymmetric branes wrapping even cycles in type IIB and odd cycles in type IIA must correspond to a correctly generalised definition of holomorphic and coisotropic branes respectively. For the cases we are interested in we can adapt the discussion presented in [10, 12] for backgrounds with only nontrivial NS fields. Furthermore we will use some notions of generalised complex geometry [3, 4] and a summary of the basic definitions needed here are provided in, for example, [1, 12, 10].

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Let us first recall that, for the general r-cycle, the second conditions of (3.14) and (3.15) come from the requirement that $\hat{\gamma}'_{(r)}(\mathcal{F})\eta^{(2)}_+$ must be parallel to $\eta^{(1)}_{(-)^r}$. It is then possible to see [10] that this condition is equivalent to

$$J_1|_\Sigma = (-)^r R J_2 R^{-1}|_\Sigma \,, \tag{4.1}$$

where J_1 and J_2 are the almost complex structures associated to the six-dimensional spinors $\eta^{(1)}_+$ and $\eta^{(2)}_+$ respectively (see appendix A for the basic definitions and properties), and the action of the rotation matrix R on $T_M|_\Sigma = T_\Sigma \oplus \mathcal{N}_\Sigma$ is as follows. If $p_{||}$ and $p_\perp$ are the projectors on the tangent and normal bundle of the brane respectively, then R acts as a reflection in the normal directions ($Rp_\perp = p_\perp R = -p_\perp$) while the action of R along T_Σ is defined by

$$p^T_{||}(g - \mathcal{F})p_{||} = p_{||}(g + \mathcal{F})p_{||}R\,, \tag{4.2}$$

where $\mathcal{F}$ is now naturally thought of as a section of $\Lambda^2 T^\star_M|_\Sigma$ such that $p^T_\perp \mathcal{F} = \mathcal{F}p_\perp = 0$. The pure spinors Ψ^+ and Ψ^- are associated to generalised almost complex structures $\mathcal{J}_+$ and $\mathcal{J}_-$ on $T_M \oplus T^\star_M$. One can prove that these can be written in terms of J_1 and J_2 as follows [4, 12, 10]:

$$\mathcal{J}_\pm = \frac{1}{2}\begin{pmatrix} J_1 \mp J_2 & (J_1 \pm J_2)g^{-1} \\ g(J_1 \pm J_2) & g(J_1 \mp J_2)g^{-1} \end{pmatrix}. \tag{4.3}$$

One can then see that (4.1) is equivalent to the following condition for $\mathcal{J}_\pm$ restricted on $T_M \oplus T^\star_M|_\Sigma$

$$\mathcal{J}_{(-)^{r+1}} = \mathcal{R}^{-1}\mathcal{J}_{(-)^{r+1}}\mathcal{R}\,, \tag{4.4}$$

where $\mathcal{R}$ acts in the following way on $T_M \oplus T^\star_M|_\Sigma$

$$\mathcal{R} = \frac{1}{2}\begin{pmatrix} r & 0 \\ \mathcal{F}r + r^T\mathcal{F} & -r^T \end{pmatrix}, \,, \tag{4.5}$$

with $r = p_{||} - p_\perp$.

The D-brane worldvolume wrapping the internal cycle Σ, specified by the couple $(\Sigma, \mathcal{F})$ where $\mathcal{F}$ is such that $d\mathcal{F} = P_\Sigma[H]$, can be seen as a generalised submanifold as defined by Gualtieri in [4]. Gualtieri also defines a *generalised tangent bundle* $\tau^{\mathcal{F}}_\Sigma$ associated to the brane. The key point is that the elements $X \in T_M \oplus T^\star_M|_\Sigma$ belonging to $\tau^{\mathcal{F}}_\Sigma$ can be characterised by the condition [12]

$$\mathcal{R}X = X\,. \tag{4.6}$$

The subsequent step is to remember that, given an (integrable) generalised complex structure $\mathcal{J}$ on M, Gualtieri defines a *generalised complex submanifold* as a generalised submanifold $(\Sigma, \mathcal{F})$ with generalised tangent bundle $\tau^{\mathcal{F}}_\Sigma$ stable under $\mathcal{J}$.

From (4.4) and (4.6) we arrive at the conclusion that the second conditions in (3.14) and (3.15) are each equivalent to the requirement that *supersymmetric D-branes wrapping even-cycles in type IIB and odd-cycles in type IIA must be generalised complex submanifolds with respect to the (integrable) generalised complex structures $\mathcal{J}_-$ and $\mathcal{J}_+$ respectively.* These generalised complex submanifolds can be seen as the most natural generalisation of complex cycles with $\mathcal{F}$ of kind $(1,1)$ in type IIB and of coisotropic cycles in type IIA [27, 4].

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5. D-branes on SU(3)-structure manifolds

In this section we pause the discussion of general $SU(3) \times SU(3)$ structure manifolds to comment on the SU(3) structure subcase. We recall that this is obtained when we can write $\eta_+^{(1)} = a\eta_+$ and $\eta_+^{(2)} = b\eta_+$ ($\eta_+^\dagger \eta_+ = 1$), remembering that in order to have supersymmetric branes we have to fulfil the necessary condition $|a| = |b|$. In this case the pure spinors $\Psi^\pm$ can be defined in terms of the almost complex structure J and the $(3,0)$-form Ω associated to η_+ as explained in appendix A. Using the Fierz decomposition it is possible to show that

$$\eta_\pm \otimes \eta_\pm^\dagger = \frac{1}{8} e^{\mp i \not{J}} \,, \qquad \eta_+ \otimes \eta_-^\dagger = -\frac{i}{8} \not{\Omega} \,. \tag{5.1}$$

We immediately see that in the SU(3)-structure case Ψ^+ and Ψ^- reduce to

$$\Psi^+ = \frac{a\bar{b}}{8} e^{-iJ} \,, \qquad \Psi^- = -\frac{iab}{8} \Omega \,. \tag{5.2}$$

Since we must require that $|a| = |b|$, we can pose

$$\frac{a}{b} \equiv e^{i\phi} \,, \qquad \frac{a}{b^*} \equiv e^{i\tau} \,, \tag{5.3}$$

and the supersymmetry conditions for the wrapped branes now read

$$\begin{aligned} \Big\{ \mathrm{Im}\,(ie^{i\phi} P[e^{-iJ}]) \wedge e^{\mathcal{F}} \Big\}_{(2k)} &= 0 \,, \\ \Big\{ P[dx^m \wedge \Omega + g^{mn} \imath_n \Omega] \wedge e^{\mathcal{F}} \Big\}_{(2k)} &= 0 \,, \end{aligned} \tag{5.4}$$

for even $2k$-cycles, and

$$\begin{aligned} \Big\{ \mathrm{Im}\,(e^{i\tau} P[\Omega]) \wedge e^{\mathcal{F}} \Big\}_{(2k+1)} &= 0 \,, \\ \Big\{ P[dx^m \wedge e^{iJ} + g^{mn} \imath_n e^{iJ}] \wedge e^{\mathcal{F}} \Big\}_{(2k+1)} &= 0 \,, \end{aligned} \tag{5.5}$$

for odd $(2k+1)$-cycles. Again, these conditions really imply that it is possible to choose an orientation on the D-brane in order for it to be supersymmetric and generally reversing the orientation does not preserve supersymmetry. As in the general case, they can be substituted by the following equivalent conditions which also provide the necessary requirement on the orientation

$$\Big\{ \mathrm{Re}\,(-ie^{i\phi} P[e^{-iJ}]) \wedge e^{\mathcal{F}} \Big\}_{(2k)} = \sqrt{\det(P[g] + \mathcal{F})} d\sigma^1 \wedge \cdots \wedge d\sigma^{2k} \,, \tag{5.6}$$

for even $2k$-cycles, and

$$\Big\{ \mathrm{Re}\,(-e^{i\tau} P[\Omega]) \wedge e^{\mathcal{F}} \Big\}_{(2k+1)} = \sqrt{\det(P[g] + \mathcal{F})} d\sigma^1 \wedge \cdots \wedge d\sigma^{2k+1} \,, \tag{5.7}$$

for odd $(2k+1)$-cycles. Note that in the SU(3)-case type IIB and IIA backgrounds have complex and symplectic internal manifolds respectively. The above conditions have the same form as those derived in [7] for branes with nontrivial worldvolume fluxes on spaces

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with no fluxes, and can be seen as their natural generalisation (see also the discussion in [8] for the type IIB case). In particular, from the discussion of the previous section, the second conditions in (5.4) and (5.5) now require that supersymmetric branes are complex branes with $(1,1)$ field strength $\mathcal{F}$ in type IIB and coisotropic branes of the kind discussed in [27] in type IIA (see [4, section 7.2]). Also, the above conditions are obviously exchanged by the generalised mirror symmetry, that in this case takes the form

$$e^{i\phi}e^{-iJ} \leftrightarrow -ie^{i\tau}\Omega\,. \tag{5.8}$$

6. Generalised calibrations for $\mathcal{N}=1$ vacua

We shall now proceed to discuss the meaning of the supersymmetry conditions in the general SU(3) $\times$ SU(3) case. We will see how the conditions in the form (3.16) and (3.17) can be interpreted as generalised calibration conditions. Then the first of each pair of conditions (3.14) and (3.15) encodes the necessary requirement related to the stability of the supersymmetric D-brane that must be added to the geometrical characterisation given in section 4.

Let us first of all introduce the appropriate definition of generalised calibration for the general class of $\mathcal{N}=1$ manifolds we are considering, starting from the supersymmetry conditions for four-dimensional space-time filling branes derived in the previous sections. We will see how it is possible to naturally introduce a generalised calibration that minimises the energy and with respect to which supersymmetric cycles are calibrated. The notion of generalised calibration was first introduced in [28] to describe supersymmetric branes on backgrounds with fluxes, and studied in several subsequent papers (see for example [29, 30]). The idea is that the calibration should minimise the brane energy which does not necessarily coincide with the volume wrapped by the brane. It has been shown in [10] how, in the case of pure NS supersymmetric backgrounds, it is possible to introduce another notion of generalised calibration which naturally takes into account the role of the worldvolume field strength f. We will now see how an analogous definition of generalised calibration can also be used for general $\mathcal{N}=1$ backgrounds with nontrivial RR fluxes.

We define a *generalised calibration* as a sum of forms of different degree $\omega=\sum_k \omega_{(k)}$ such that $d_H\omega=(d+H\wedge)\omega=0$ and

$$P_\Sigma[\omega]\wedge e^{\mathcal{F}} \leq \mathcal{E}(\Sigma,\mathcal{F})\,, \tag{6.1}$$

for any D-brane $(\Sigma,\mathcal{F})$ characterised by the wrapped cycle Σ and the worldvolume field strength $\mathcal{F}$ and with energy density $\mathcal{E}$.[6] In (6.1) and all other expressions in this section involving sums of forms of different degree on the cycle wrapped by the brane, we understand that only forms of rank equal to the dimension of the cycle are selected. Furthermore, the inequalities between these forms refer to the associated scalar components in the one-dimensional base given by the standard (oriented) volume form.

[6] This definition is completely equivalent to the definition used in [10] where a generalised calibration $\tilde\omega$ is closed, i.e. $d\tilde\omega=0$, and satisfies the relation $P_\Sigma[\omega]\wedge e^{f} \leq \mathcal{E}(\Sigma,\mathcal{F})$. The two generalised calibrations are obviously related by $\tilde\omega=\omega\wedge e^{B}$. We prefer our choice as it involves the worldvolume field-strength f only through the gauge invariant combination $\mathcal{F}$.

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A D-brane $(\Sigma, \mathcal{F})$ is then *calibrated in a generalised sense* by $\omega = \sum_k \omega_{(k)}$, if it satisfies the condition

$$P_\Sigma[\omega] \wedge e^{\mathcal{F}} = \mathcal{E}(\Sigma, \mathcal{F})\,. \tag{6.2}$$

Since the generalised calibration ω is d_H-closed, one can immediately prove that the saturation of the calibration bound is a minimal energy condition. Let $E(\Sigma, \mathcal{F})$ be the four-dimensional energy density of a calibrated wrapped D-brane $(\Sigma, \mathcal{F})$. Consider a continuous deformation to a different brane configuration $(\Sigma', \mathcal{F}')$ such that we can take a chain $\mathcal{B}$ and a field-strength $\hat{\mathcal{F}}$ on it (with $d\hat{\mathcal{F}} = P_{\mathcal{B}}[H]$), such that $\partial\mathcal{B} = \Sigma - \Sigma'$ and the restriction of $\hat{\mathcal{F}}$ to Σ and Σ' gives $\mathcal{F}$ and $\mathcal{F}'$ respectively. We then have

$$\begin{aligned} E(\Sigma, \mathcal{F}) &= \int \mathcal{E}(\Sigma, \mathcal{F}) = \int_\Sigma P[\omega] \wedge e^{\mathcal{F}} \\ &= \int_{\mathcal{B}} P[d_H\omega] \wedge e^{\hat{\mathcal{F}}} + \int_{\Sigma'} P[\omega] \wedge e^{\mathcal{F}'} \\ &= \int_{\Sigma'} P[\omega] \wedge e^{\mathcal{F}'} \leq \int \mathcal{E}(\Sigma', \mathcal{F}') = E(\Sigma', \mathcal{F}')\,. \end{aligned} \tag{6.3}$$

A calibration condition can then be seen as a stability condition for a D-brane under continuous deformations.

We will now see how the first supersymmetry conditions in (3.16) and (3.17) can be rephrased as generalised calibration conditions. In order to prove this, we have to construct the generalised calibration appropriate to our case. Let us start by recalling that we are restricting to the case in which $\eta^{(1)}$ and $\eta^{(2)}$ have the same norm. Then the standard Schwarz inequality

$$||i\hat{\gamma}'_{(r)}(\mathcal{F})\eta^{(2)}_+ + \eta^{(1)}_{(-)^r}|| \leq ||i\hat{\gamma}'_{(r)}(\mathcal{F})\eta^{(2)}_+|| + ||\eta^{(1)}_{(-)^r}||\,, \tag{6.4}$$

implies that we have the following completely general inequalities

$$\mathrm{Re}\,[i\eta^{(1)\dagger}_+\hat{\gamma}'_{(2k)}(\mathcal{F})\eta^{(2)}_+] \leq |a|^2\,, \qquad \mathrm{Re}\,[i\eta^{(1)\dagger}_-\hat{\gamma}'_{(2k+1)}(\mathcal{F})\eta^{(2)}_+] \leq |a|^2\,, \tag{6.5}$$

which, remembering (3.9), are clearly saturated when we are considering supersymmetric cycles. Using expression (3.5) for $\hat{\gamma}'_{(r)}$ it is not difficult to see that from these relations we obtain the conditions

$$\begin{aligned} \Big\{\mathrm{Re}\,(-iP[\Psi^+]) \wedge e^{\mathcal{F}}\Big\}_{(2k)} &\leq \frac{|a|^2}{8}\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1 \wedge \cdots \wedge d\sigma^{2k}\,, \\ \Big\{\mathrm{Re}\,(-iP[\Psi^-]) \wedge e^{\mathcal{F}}\Big\}_{(2k+1)} &\leq \frac{|a|^2}{8}\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1 \wedge \cdots \wedge d\sigma^{2k+1}\,. \end{aligned} \tag{6.6}$$

Once we impose that the D-branes must wrap generalised complex submanifolds in M, one sees that requiring the inequalities in (6.6) to be saturated is equivalent to requiring that the D-branes we are considering satisfy the supersymmetry conditions (3.16) and (3.17).

We would now like to use these inequalities to construct a generalised calibration for this space-time filling branes. Given the RR field-strength ansatz specified in (2.3), we can analogously decompose the RR potentials in the following way

$$C_{(n)} = \hat{C}_{(n)} + dx^0 \wedge \ldots \wedge dx^3 \wedge e^{4A}\tilde{C}_{(n-4)}\,, \tag{6.7}$$

and then express the internal RR field strengths in terms of the internal RR potentials

$$\begin{aligned}\hat{F}_{(k+1)} &= d\hat{C}_{(k)} + H \wedge \hat{C}_{(k-2)} , \\ \tilde{F}_{(k+1)} &= d\tilde{C}_{(k)} + H \wedge \tilde{C}_{(k-2)} + 4dA \wedge \tilde{C}_{(k)} . \end{aligned} \tag{6.8}$$

Our space-time filling branes couple only to the "tilded" RR fields. Since we are considering static configurations, we can extract from the Dirac-Born-Infield plus Chern-Simons action the following effective energy density for a space-time filling brane wrapping an internal n-cycle

$$\mathcal{E} = e^{4A}\Big\{e^{-\Phi}\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1 \wedge \cdots \wedge d\sigma^n - \Big(\sum_k P[\tilde{C}_{(k)}] \wedge e^{\mathcal{F}}\Big)_{(n)}\Big\} , \tag{6.9}$$

where for simplicity we have omitted the overall factor given by the D-brane tension. We can now write the inequalities (6.6) in terms of a lower bound on the energy density

$$P[\omega] \wedge e^{\mathcal{F}} \leq \mathcal{E} , \tag{6.10}$$

where we have used the sum of forms of different degrees $\omega = \sum_k \omega_{(k)}$ given by

$$\begin{aligned}\omega_{IIA} &= e^{4A}\left[\mathrm{Re}\left(\frac{-8i}{|a|^2}e^{-\Phi}\Psi^-\right) - \sum_k \tilde{C}_{(2k+1)}\right] , \\ \omega_{IIB} &= e^{4A}\left[\mathrm{Re}\left(\frac{-8i}{|a|^2}e^{-\Phi}\Psi^+\right) - \sum_k \tilde{C}_{(2k)}\right] . \end{aligned} \tag{6.11}$$

Note that in the left hand side of (6.10) one can completely factorise the contributions of the background quantities through the pullback on the cycle of ω and B, and the contribution from the worldvolume field-strength f.

It is clear from (6.10) that the ω's defined in (6.11) represent a good candidate for generalised calibrations as described at the beginning of this section. To prove that this is indeed the case, it remains to show that the ω's in (6.11) are d_H-closed. In order to do this, it will be enough to use the equations (2.15) and (2.18), which characterise our $\mathcal{N} = 1$ backgrounds.

Let us impose the vanishing of the d_H-differential of the ω's defined in (6.11). This gives the following condition to have properly defined calibrations

$$\begin{aligned} d_H\omega_{IIA} = 0 \Leftrightarrow [d + (H + 4dA)\wedge]\left[\frac{1}{|a|^2}e^{-\Phi}\,\mathrm{Re}\left(i\Psi^-\right)\right] &= -\frac{1}{8}\sum_{k=0,1,2,3}\tilde{F}_{(2k)} , \\ d_H\omega_{IIB} = 0 \Leftrightarrow [d + (H + 4dA)\wedge]\left[\frac{1}{|a|^2}e^{-\Phi}\,\mathrm{Re}\left(i\Psi^+\right)\right] &= -\frac{1}{8}\sum_{k=0,1,2}\tilde{F}_{(2k+1)} . \end{aligned} \tag{6.12}$$

One immediately sees that the conditions (2.15) and (2.18) imply that the above requirements are indeed satisfied. This concludes our proof that our $\mathcal{N} = 1$ backgrounds are generalised complex manifolds with generalised calibrations defined in (6.11), such that supersymmetric four-dimensional spacetime filling branes wrap generalised complex submanifolds, which are also generalised calibrated.

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One can get more intuition on the structure of the above generalised calibrations by considering the SU(3)-structure subcase. The generalised calibrations then take the form

$$\begin{aligned}\omega_{IIA} &= e^{4A}\Big[\mathrm{Re}\Big(-e^{i\tau}e^{-\Phi}\Omega\Big)-\sum_k \tilde{C}_{(2k+1)}\Big]\,,\\ \omega_{IIB} &= e^{4A}\Big[\mathrm{Re}\Big(-ie^{i\phi}e^{-\Phi}e^{-iJ}\Big)-\sum_k \tilde{C}_{(2k)}\Big]\,.\end{aligned} \tag{6.13}$$

We then explicitly see how these calibrations generalise the usual calibrations in Calabi-Yau spaces through crucial modifications introduced by the nontrivial dilaton, warp-factor and fluxes.

Note also that the generalised calibrations (6.11) are naturally related by the mirror symmetry (2.19), if we exchange $\sum_k \tilde{C}_{(2k)}$ and $\sum_k \tilde{C}_{(2k+1)}$. These sums can be seen as H-twisted potentials of the sums of internal field strengths $\tilde{F}_A$ and $\tilde{F}_B$ as defined in (2.16) and (2.17). If we think in terms of untwisted quantities we then get a mirror symmetry for the potentials of the form

$$\sum_k \tilde{C}_{(2k)}\wedge e^B \leftrightarrow \sum_k \tilde{C}_{(2k+1)}\wedge e^B\,, \tag{6.14}$$

which clearly recalls the form of the transformation rules of the RR-potentials under T-duality.

Let us observe that the generalised calibration ω defined above is a sum of forms which are not generally globally defined, since they are not invariant under the RR gauge transformations. Indeed, consider the gauge transformation

$$\sum_n \delta\tilde{C}_{(n)} = e^{-4A}d_H\lambda\,, \tag{6.15}$$

preserving the decomposition (6.7), where λ is a sum of even (odd) forms for type IIA (IIB). Then ω transforms as $\omega\to\omega-d_H\lambda$, since it is related to the D-brane energy density which naturally depends on the RR gauge potentials. As an alternative, we could also introduce an equivalent globally defined generalised calibration $\hat{\omega}=\sum_n\hat{\omega}_{(n)}$ which is more in the spirit of that adopted in [28]. First, in our class of backgrounds, $\hat{\omega}$ is no longer d_H closed, but must satisfy the condition

$$d_H\hat{\omega} = e^{4A}\sum_k \tilde{F}_{(k)}\,. \tag{6.16}$$

Secondly, the energy density minimisation condition (6.10) is replaced by the condition

$$P_\Sigma[\hat{\omega}]\wedge e^{\mathcal{F}} \le e^{4A-\Phi}\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1\wedge\cdots d\sigma^n\,, \tag{6.17}$$

for any D-brane $(\Sigma,\mathcal{F})$ wrapping an internal n-dimensional cycle. It is clear from our previous discussion that such an alternative generalized calibration is given by

$$\hat{\omega} = -\frac{8e^{4A-\Phi}}{|a|^2}\,\mathrm{Re}(i\Psi)\,, \tag{6.18}$$

where $\Psi = \Psi^+$ for type IIB and $\Psi = \Psi^-$ for type IIA. We obviously have that $\omega = \hat{\omega} - e^{4A}\sum_n \tilde{C}_{(n)}$ and the alternative generalized calibration $\hat{\omega}$ can be essentially identified with the imaginary part of the non-integrable pure spinor characterising the $\mathcal{N} = 1$ background considered.

As we are assuming $|a| = |b|$, the condition (6.16) is equivalent to the imaginary part of the first of the background supersymmetry conditions (2.15), thus giving a physical interpretation for it. It is nice to note that an analogous conclusion can be reached for the remaining equations in (2.15). Indeed, we have seen in section 3 how we could also consider supersymmetric branes filling only two or three flat space-time directions, giving rise to an effective string or domain wall respectively with appropriately chosen phases α in (3.6). One can then repeat the arguments of this section for these cases, with the generalised calibrations now be given by

$$\omega^{(string)} = \frac{8e^{2A-\Phi}}{|a|^2}\,\mathrm{Re}(\Psi_1)\,, \qquad \omega^{(DW)} = \frac{8e^{3A-\Phi}}{|a|^2}\,\mathrm{Re}(e^{i\theta}\Psi_2)\,, \tag{6.19}$$

where $\Psi_1 = \Psi^+$ (Ψ^-) and $\Psi_2 = \Psi^-$ (Ψ^+) for type IIB (IIA), and θ is an arbitrary (constant) phase. The generalised calibrations $\omega^{(string)}$ and $\omega^{(DW)}$ now satisfy the condition (6.17) with e^{4A} substituted by e^{2A} and e^{3A} respectively. Furthermore, they must now be d_H-closed, since the coupling to the background RR-fields vanishes for these configurations . It is then easy to see that the condition $d_H\omega^{(string)} = 0$ is equivalent to the real part of the first of (2.15) (with $|a| = |b|$), while $d_H\omega^{(DW)} = 0$ for any θ is equivalent to the second of (2.15) . We then see how, in the subcase where the two internal spinors have the same norm, the background supersymmetry conditions (2.15) have a physical interpretation as conditions for the existence of generalised calibrations for the allowed supersymmetric D-brane configurations. This correspondence between background supersymmetry conditions and generalized calibrations has been extensively discussed in [29] and we see here how it works perfectly in the cases we have considered.

7. Conclusions

In this paper we have studied the conditions for having supersymmetric D-branes in type II backgrounds with general NS and RR fields preserving four-dimensional Poincaré invariance and $\mathcal{N} = 1$ supersymmetry, focusing on D-branes filling the four flat directions. It turns out that the supersymmetry conditions for D-branes obtained from κ-symmetry can be elegantly expressed in terms of the two pure spinors that define the $\mathrm{SU}(3)\times\mathrm{SU}(3)$-structure on the internal six-dimensional manifold. We have shown that the supersymmetry conditions give two important pieces of information on the supersymmetric D-branes, regarding the geometry and the stability of the branes, as happens in absence of fluxes, and involving the two pure spinors separately.

Firstly, the D-brane must wrap a generalised complex submanifold defined with respect to the integrable generalised complex structure of the internal manifold. This can be introduced thanks to the integrability of one of the two pure spinors coming from the requirement of $\mathcal{N} = 1$ supersymmetry. The SU(3) structure subcase provides a clear example where this condition means that the brane must wrap a holomorphic cycle with $(1,1)$ field strength $\mathcal{F}$ in type IIB and a coisotropic cycle of the kind discussed in [27, 4] in type IIA. In the more general $\mathrm{SU}(3)\times\mathrm{SU}(3)$ case the equivalent type IIA/IIB identifications become slightly mixed.

Secondly, on the wrapped internal n-cycle one must furthermore impose a condition of the form $\{\mathrm{Im}(P[i\Psi])\wedge\mathcal{F}\}_{(n)}=0$, where Ψ is the non-integrable pure spinor. This condition is related to the stability of the D-brane. Note that it is the non-integrable pure spinor that now plays the relevant role and the fact that it should be connected to some dynamical information for the D-branes can be linked to the role of the nontrivial RR-fields as obstructions to the integrability of the pure spinor. Then, a supersymmetric D-brane configuration must satisfy the above two conditions, plus an appropriate choice of its orientation which is in general not arbitrary due to presence of nontrivial background RR fields.

The above requirements that characterise supersymmetric D-branes are equivalent to the condition that the D-brane must be calibrated in a generalised sense with respect to an appropriate definition of generalised calibration. This encodes a requirement of minimisation of the energy of the brane and involves the non-integrable pure spinor. The non-integrability of this pure spinor is due to the non-vanishing RR-fields, which also couple to D-branes and so must enter the associated generalised calibrations. Then one sees that the non-integrability of the pure spinor is exactly what is needed to compensate for the presence of the RR terms in the generalised calibration in order for it to be well defined. This strict relation between the non-integrable pure spinor and a generalised calibration can be made even more explicit by using the equivalent alternative definition of generalised calibration given in (6.16) and (6.17). Furthermore, as we discuss at the end of section 6, by considering D-branes filling only two or three flat directions, the conditions for the existence of well defined calibrations associated to supersymmetric D-branes are completely equivalent to the background supersymmetry conditions (2.15), thus giving a clear physical interpretation for them.

To conclude, it is intriguing to see how the two pure spinors can be fruitfully used in the description of the geometrical and stability features of supersymmetric D-branes. Also, all the results discussed in this paper confirm the interpretation of the symmetry (2.19) relating type IIA and IIB backgrounds as a generalised mirror symmetry, exchanging also odd and even dimensional supersymmetric cycles and the corresponding generalised calibrations. These results may hide some deeper insight into the understanding of string theory on general backgrounds with fluxes and its relation to generalised geometry.

Acknowledgments

We would like to thank G. Bonelli, M. Graña, F. Marchesano, J. Sparks, K. Stelle and A. Uranga. We especially thank A. Van Proeyen for his comments and careful reading of the manuscript. This work is supported in part by the Federal Office for Scientific, Technical and Cultural Affairs through the "Interuniversity Attraction Poles Programme

— Belgian Science Policy" P5/27 and by the European Community's Human Potential Programme under contract MRTN-CT-2004-005104 'Constituents, fundamental forces and symmetries of the universe'. PS is supported by the Leverhulme Trust.

A. Basic definitions for SU(3) structure manifolds

In this section we will review some basic facts about an SU(3)-structure manifold M, that is characterised by the existence of a globally defined spinor η_+, such that $||\eta||^2 = |a|^2$ (for a nice review on this subject see for example [31]). This spinor allows one to introduce an associated almost complex structures with respect to which the six-dimensional metric g_{mn} is hermitean. For our purposes the most useful choice is given by

$$J_{mn} = -\frac{i}{|a|^2}\eta_+^\dagger \hat{\gamma}_{mn}\eta_+ \,. \tag{A.1}$$

Using the Fierz identities, it is possible to show that

$$J_m{}^p J_p{}^n = -\delta_m^n \,, \qquad J_m{}^p J_n{}^q g_{pq} = g_{mn} \,. \tag{A.2}$$

This almost complex structure allows one to introduce the projector on holomorphic indices

$$\mathcal{P}_m{}^n = \frac{1}{2}(\delta_m^n - iJ_m{}^n) \,, \tag{A.3}$$

and the associated anti-holomorphic projector $\bar{\mathcal{P}}_m{}^n = (\mathcal{P}_m{}^n)^*$. One can then split r-forms in (p,q)-forms, with $p+q=r$, in the standard way.

The following relations hold

$$\eta_+^\dagger \hat{\gamma}^m \hat{\gamma}_n \eta_+ = 2|a|^2 \bar{\mathcal{P}}^m{}_n \,, \qquad \eta_-^\dagger \hat{\gamma}^m \hat{\gamma}_n \eta_- = 2|a|^2 \mathcal{P}^m{}_n \,. \tag{A.4}$$

Then $\hat{\gamma}_m \eta_+ = \mathcal{P}_m{}^n \hat{\gamma}_n \eta_+$ (it is of the kind $(1,0)$ in the index m), and the base (3.10) is indeed eight dimensional. The general six-dimensional Dirac spinor χ can then be decomposed as

$$\chi = \lambda_1 \eta_+ + \lambda_2 \eta_- + \xi_1^m \hat{\gamma}_m \eta_+ + \xi_2^m \hat{\gamma}_m \eta_- \,, \tag{A.5}$$

where ξ_1^m is a $(1,0)$-vector ($\mathcal{P}_m{}^n \xi_1^m = \xi_1^n$) and ξ_2^m is a $(0,1)$-vector ($\bar{\mathcal{P}}_m{}^n \xi_1^m = \xi_1^n$). Then,

$$\begin{aligned}
\lambda_1 &= \frac{1}{|a|^2}\eta_+^\dagger \chi \,, \qquad \lambda_2 = \frac{1}{|a|^2}\eta_-^\dagger \chi \,, \\
\xi_1^m &= \frac{1}{2|a|^2}\eta_+^\dagger \hat{\gamma}^m \chi \,, \qquad \xi_2^m = \frac{1}{2|a|^2}\eta_-^\dagger \hat{\gamma}^m \chi \,.
\end{aligned} \tag{A.6}$$

Analogously to the CY_3 case, we can also introduce a $(3,0)$ form Ω defined by

$$\Omega_{mnp} = -\frac{i}{a^2}\eta_-^\dagger \hat{\gamma}_{mnp}\eta_+ \,. \tag{A.7}$$

By applying Fierz identities it is possible to see that

$$\frac{1}{3!}J \wedge J \wedge J = \frac{i}{8}\Omega \wedge \bar{\Omega} \,, \qquad J \wedge \Omega = 0 \,, \tag{A.8}$$

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as for Calabi-Yau manifolds. The existence of a globally defined non-degenerate (real) J and a globally defined non-degenerate (complex) Ω satisfying the conditions (A.8) actually characterises SU(3)-structure manifolds. In our case we are considering the more general case of internal manifolds M with SU(3)$\times$SU(3)-structure group for $T_M \oplus T^\star_M$. This contains as subcases the SU(3)-structure manifolds case and the even more restricted manifolds with SU(2)-structure, that contain two different independent SU(3) structures and requires the vanishing of the Euler characteristic χ of M.

B. Background supersymmetry conditions

Here we recall the way to derive the background supersymmetry conditions as given in (2.15). In [2] it was first shown how the Killing equations can be written in this elegant form in terms of the pure spinors $\Psi^\pm$. However, repeating the calculation we find a slightly different result from that given in [2]. For this reason we present some details of the calculation leading to (2.15).

We will follow the method described in [1, 2] which uses the democratic formalism of [32], and in particular we adopt their conventions in the following calculation. The result can be translated back to our convention by using the rules specified in footnote 2 on page 4. It will be sufficient to consider only the type IIA case (the type IIB case is completely analogous), for which the supersymmetry transformations for the gravitino are

$$\delta\psi_M = \nabla_M \varepsilon + \frac{1}{4} H_M \Gamma_{(10)} \varepsilon + \frac{1}{16} e^\Phi \sum_n \not{F}_{(2n)} \, \Gamma_M \Gamma^n_{(10)} \sigma_1 \, \varepsilon \,, \tag{B.1}$$

where ε is defined in (2.7), $\not\partial \equiv \hat\gamma^m \partial_m$ and $H_M \equiv \frac{1}{2} H_{MNP} \Gamma^{NP}$. The modified RR field strengths $F_{(2n)} = dC_{(2n-1)} - H \wedge C_{(2n-3)}$ are related by the conditions $F_{(2n)} = (-)^n \star_{10} F_{(10-2n)}$. We will also need the modified dilatino transformation

$$\Gamma^M \delta\psi_M - \delta\lambda = \left(\not\nabla - \not\partial \Phi + \frac{1}{4} \not{H} \Gamma_{(10)} \right) \varepsilon \,. \tag{B.2}$$

Using the 4d+6d decomposition of spinors, field strengths and gamma matrices described in section 2, we may rewrite the transformations above as conditions upon the internal spinors $\eta^{(a)}_\pm$. For the external component of the gravitino transformation $\delta\psi_\mu$ one then finds

$$\frac{1}{2} \not\partial A \eta^{(1)}_+ + \frac{e^\Phi}{16} \left(\hat{\not{F}}_{A1} + i \tilde{\not{F}}_{A1} \right) \eta^{(2)}_- = 0 \,, \tag{B.3}$$

$$\frac{1}{2} \not\partial A \eta^{(2)}_- + \frac{e^\Phi}{16} \left(\hat{\not{F}}_{A2} + i \tilde{\not{F}}_{A2} \right) \eta^{(1)}_+ = 0 \,, \tag{B.4}$$

where $F_{A1} = F_{(0)} - F_{(2)} + F_{(4)} - F_{(6)}$ and $F_{A2} = F_{(0)} + F_{(2)} + F_{(4)} + F_{(6)}$. The corresponding decomposition on the internal component $\delta\psi_m$ gives

$$\nabla_m \eta^{(1)}_+ + \frac{1}{4} H_m \eta^{(1)}_+ + \frac{e^\Phi}{16} \left(\hat{\not{F}}_{A1} - i \tilde{\not{F}}_{A1} \right) \hat\gamma_m \eta^{(2)}_- = 0 \,, \tag{B.5}$$

$$\nabla_m \eta^{(2)}_- - \frac{1}{4} H_m \eta^{(2)}_- + \frac{e^\Phi}{16} \left(\hat{\not{F}}_{A2} - i \tilde{\not{F}}_{A2} \right) \hat\gamma_m \eta^{(1)}_+ = 0 \,, \tag{B.6}$$

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and for the modified dilatino transformation we find

$$\left(\not\nabla + \frac{1}{4}\not{H} + 2\not\partial A - \not\partial\Phi\right)\eta_+^{(1)} = 0\,, \tag{B.7}$$

$$\left(\not\nabla - \frac{1}{4}\not{H} + 2\not\partial A - \not\partial\Phi\right)\eta_+^{(2)} = 0\,. \tag{B.8}$$

Consider the exterior derivative of the Clifford(6,6) spinors $\Psi^{\pm}$ in terms of bispinors, given by

$$d\Psi^{\pm} = dx^m \wedge \nabla_m \Psi^{\pm} = dx^m \wedge \left[(\nabla_m \eta_+^{(1)}) \otimes \eta_{\pm}^{(2)\dagger} + \eta_+^{(1)} \otimes (\nabla_m \eta_{\pm}^{(2)})^{\dagger}\right]. \tag{B.9}$$

We shall concentrate on $d\Psi^-$, with the aim of deriving the first expression in (2.15) for type IIA. Using the definition of Clifford(6,6) spinor representations given in [1, 2], we can rewrite (B.9) as

$$2d\Psi^- = \not\nabla\eta_+^{(1)} \otimes \eta_-^{(2)\dagger} - \eta_+^{(1)} \otimes (\not\nabla\eta_-^{(2)})^{\dagger} + \hat{\gamma}^m \eta_+^{(1)} \otimes (\nabla_m \eta_-^{(2)})^{\dagger} - \nabla_m \eta_+^{(1)} \otimes \eta_-^{(2)\dagger}\hat{\gamma}^m\,. \tag{B.10}$$

One can now use the decomposition of internal gravitino and modified dilatino supersymmetry transformations given above to evaluate the right-hand side. We now form the following bispinors from the external gravitino transformation

$$\frac{1}{2}\not\partial A \eta_-^{(1)} \otimes \eta_+^{(2)\dagger} - \frac{e^{\Phi}}{16}\left(\hat{\not{F}}_{A1} - i\tilde{\not{F}}_{A1}\right)\eta_+^{(2)} \otimes \eta_+^{(2)\dagger} = 0\,,$$

$$\frac{1}{2}\eta_-^{(1)} \otimes \eta_+^{(2)\dagger}\not\partial A - \frac{e^{\Phi}}{16}\eta_-^{(1)} \otimes \eta_-^{(1)\dagger}\left(\hat{\not{F}}_{A1} + i\tilde{\not{F}}_{A1}\right) = 0\,. \tag{B.11}$$

These two quantities can be added together with (B.10) to give the following expression,

$$\begin{aligned}
2d\Psi^- = {} & \left(\not\partial\Phi - \frac{1}{4}\not{H} - 2\not\partial A\right)\eta_+^{(1)} \otimes \eta_-^{(2)\dagger} - \eta_+^{(1)} \otimes \eta_-^{(2)\dagger}\left(\not\partial\Phi - \frac{1}{4}\not{H} - 2\not\partial A\right) - \\
& - \frac{1}{4}\hat{\gamma}^m \eta_+^{(1)} \otimes \eta_-^{(2)\dagger} H_m + \frac{1}{4} H_m \eta_+^{(1)} \otimes \eta_-^{(2)\dagger}\hat{\gamma}^m - \not\partial A \eta_-^{(1)} \otimes \eta_+^{(2)\dagger} + \eta_-^{(1)} \otimes \eta_+^{(2)\dagger}\not\partial A - \\
& - \frac{e^{\Phi}}{16}\left(\hat{\gamma}^m \eta_+^{(1)} \otimes \eta_+^{(1)\dagger}\hat{\gamma}_m + 2\eta_-^{(1)} \otimes \eta_-^{(1)\dagger}\right)(\hat{\not{F}}_{A1} + i\tilde{\not{F}}_{A1}) + \\
& + \frac{e^{\Phi}}{16}(\hat{\not{F}}_{A1} - i\tilde{\not{F}}_{A1})\left(\hat{\gamma}^m \eta_-^{(2)} \otimes \eta_-^{(2)\dagger}\hat{\gamma}_m + 2\eta_+^{(2)} \otimes \eta_+^{(2)\dagger}\right).
\end{aligned} \tag{B.12}$$

Making some manipulations, using the fact that $\hat{\gamma}_{(6)}\hat{\not{F}}_{A1} = -i\tilde{\not{F}}_{A1}$ and going back to the notations used in this paper as specified in the footnote 2, the Clifford map allows one to write this equation in the form (2.15).

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